# Rotational-Linear Parallels

# Reduced Mass

# Proton-Proton Cycle

The fusion of hydrogen in lower temperature stars like our Sun involve the following reactions yielding positrons, neutrinos, and gamma rays.

$$ {}^1_1H + {}^1_1H \rightarrow {}^2_1H + {}^0_1e^+ + \nu $$

$$ {}^1_1H + {}^2_1H \rightarrow {}^3_2He + \gamma $$

The solar neutrino problem

which can be followed by either

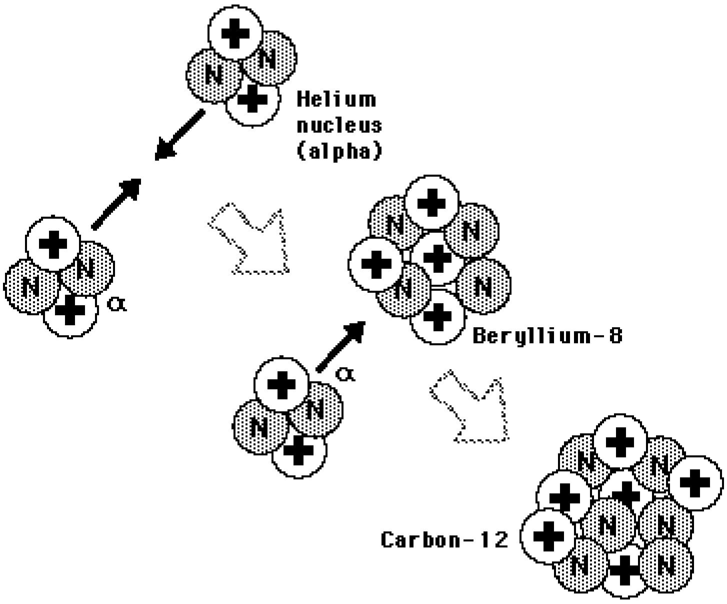
$$ {}^1_1H + {}^3_2He \rightarrow {}^4_2He + {}^0_1e^+ + \nu \quad \text{or} \quad {}^3_2He + {}^3_2He \rightarrow {}^4_2He + {}^1_1H + {}^1_1H $$

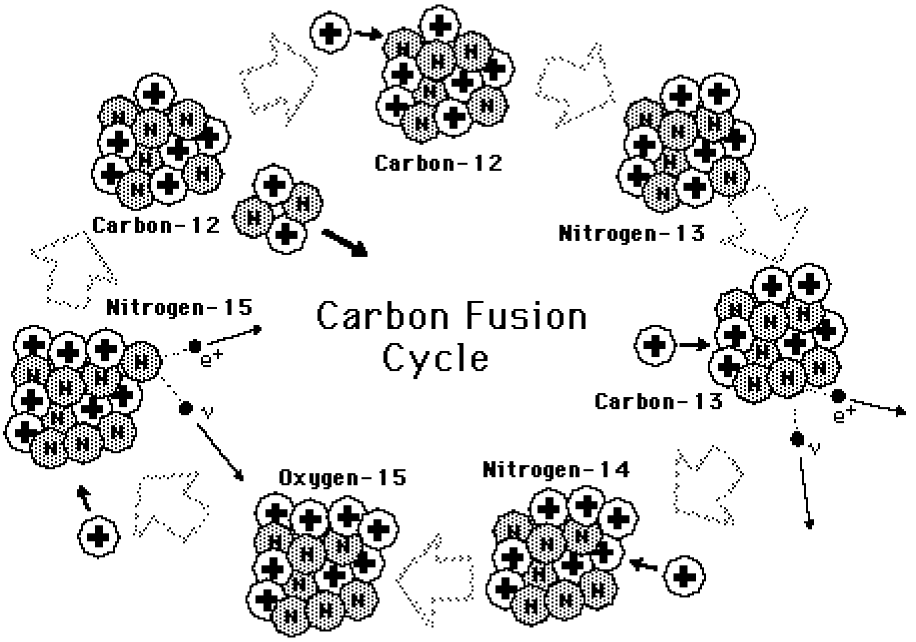
The latter of these reactions is part of what is usually called the proton-proton cycle, which yields about 25 MeV and can be combined to the form

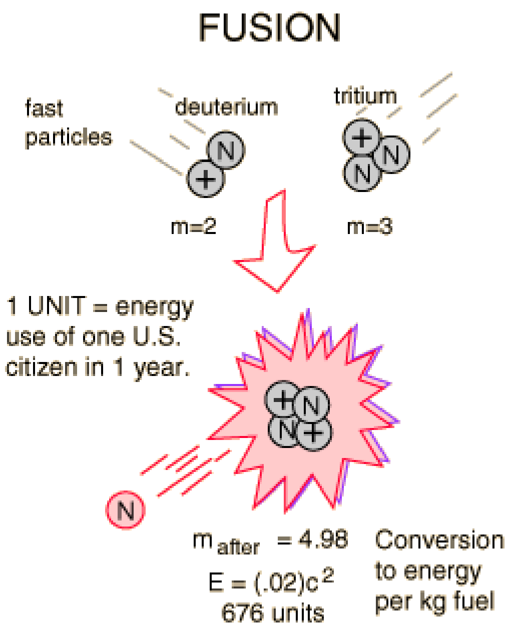
$$ 4\,{}^1_1H \rightarrow He^4_2 + 2\,{}^0_1e^+ + 2\nu $$

# Triple Alpha Process

If the central temperature of a star exceeds 100 million Kelvins, as may happen in the later phase of red giants and red supergiants, then helium can fuse to form beryllium and then carbon.


Helium
nucleus
(alpha)
α
Beryllium-8
α
Carbon-12


Carbon-12
Carbon-12
Nitrogen-13
Carbon Fusion
Cycle
Nitrogen-15
e+
ν
Carbon-13
e+
ν
Oxygen-15
Nitrogen-14


FUSION
fast
particles
deuterium
tritium
m=2
m=3
1 UNIT = energy
use of one U.S.
citizen in 1 year.
m after = 4.98
E = (.02)c2
676 units
Conversion
to energy
per kg fuel

$$pn + pn^2 = p^2n^2 + n$$

# Hydrogen Fusion Reactions

Even though a lot of energy is required to overcome the Coulomb barrier and initiate hydrogen fusion, the energy yields are enough to encourage continued research. Hydrogen fusion on the earth could make use of the reactions:

$$\left.\begin{array}{l} {}^2_1H + {}^2_1H \rightarrow {}^3_2He + {}^1_0n + 3.27\ \mathrm{MeV} \\ {}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H + 4.03\ \mathrm{MeV} \end{array}\right\} \text{deuterium-deuterium fusion}$$

$${}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0n + 17.59\ \mathrm{MeV} \quad \text{deuterium-tritium fusion}$$

deuterium cycle:

$${}^2_1H + {}^3_2He \rightarrow He^4_2 + H^1_1 + 18.3\ \mathrm{MeV}$$

.-.-.-

# Deuterium Cycle of Fusion

The four fusion reactions which can occur with deuterium can be considered to form a deuterium cycle. The four reactions:

$${}^2_1H + {}^2_1H \rightarrow {}^3_2He + {}^1_0n + 3.3\ \mathrm{MeV}$$

$${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H + 4.0\ \mathrm{MeV}$$

$${}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0n + 17.6\ \mathrm{MeV}$$

$${}^2_1H + {}^3_2He \rightarrow He^4_2 + H^1_1 + 18.3\ \mathrm{MeV}$$

can be combined as

$$6\,{}^2_1H + {}^3_1H + {}^3_2He \rightarrow 2\,{}^4_2He + {}^3_2He + {}^3_1H + 2\,{}^1_1H + 2\,{}^1_0n + 43.2\ \mathrm{MeV}$$

or

$$6\,{}^2_1H \rightarrow 2\,{}^4_2He + 2\,{}^1_1H + 2\,{}^1_0n + 43.2\ \mathrm{MeV}$$

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Fusion concepts

# Deuterium Cycle of Fusion

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# Tritium Breeding

Deuterium-Tritium fusion is the most promising of the hydrogen fusion reactions, but no tritium occurs in nature since it has a 10 year half-life. The most promising source of tritium seems to be the breeding of tritium from lithium-6 by neutron bombardment with the reaction

$${}^6_3Li + {}^1_0n \rightarrow {}^4_2He + {}^3_1H + 4.8\ MeV$$

which can be achieved by slow neutrons. This would occur if lithium were used as the coolant and heat transfer medium around the reaction chamber of a fusion reactor. Lithium-6 makes up 7.4% of natural lithium. While this constitutes a sizable supply, it is the limiting resource for the D-T process since the supply of deuterium fuel is virtually unlimited. With fast neutrons, tritium can be bred from the more abundant Li-7:

$${}^1_0n(fast) + {}^7_3Li \rightarrow {}^3_1H + {}^4_2He + {}^1_0n(slow)$$

.-.-.-.-

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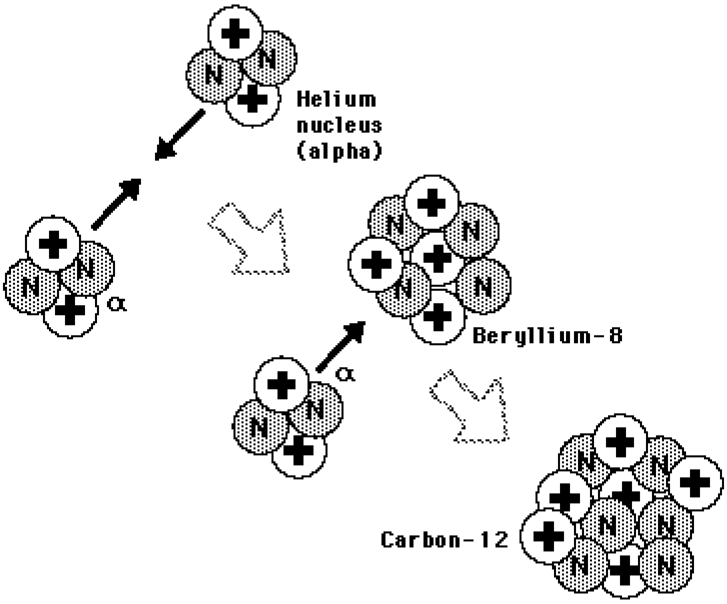


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$$^{1}_{0}\text{n(fast)} + ^{7}_{3}\text{Li} \rightarrow ^{3}_{1}\text{H} + ^{4}_{2}\text{He} + ^{1}_{0}\text{n(slow)}$$

.-.-.-.-

quasar

$$z = \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} - 1$$

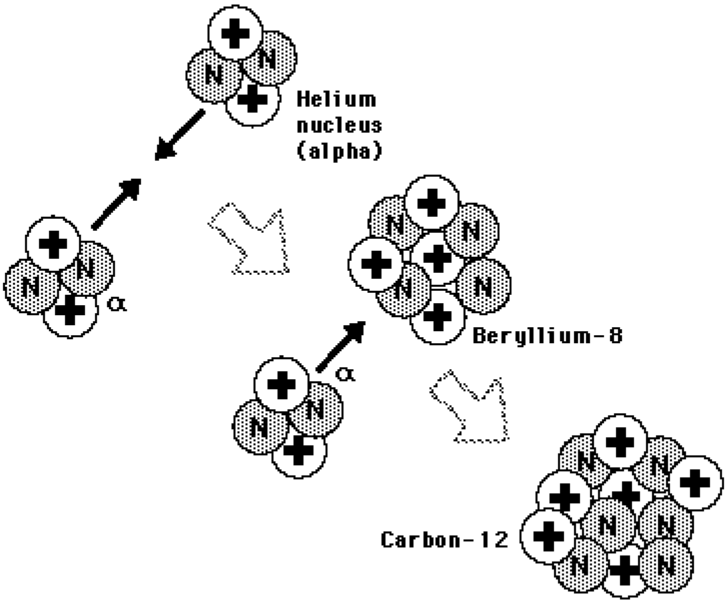
**For z =**

**v/c =**

and the range for > 100 observed quasars is z = 0.16 to 3.53. Calculation of v/c gives

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

$$^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e$$




# Charge Conjugation

Associated with the conservation laws which govern the behavior of physical particles, charge conjugation (C), parity (P) and time reversal (T) combine to constitute a fundamental symmetry called CPT invariance.

Classically, charge conjugation may seem like a simple idea: just replace positive charges by negative charges and vice versa. Since electric and magnetic fields have their origins in charges, you also must reverse these fields.

In quantum mechanical systems, charge conjugation has some further implications. It also involves reversing all the internal quantum numbers like those for lepton number, baryon number and strangeness. It does not affect mass, energy, momentum or spin.

Thinking of charge conjugation as an operator, C, then electromagnetic processes are invariant under the C operation since Maxwell's equations are invariant under C. This restricts some kinds of particle processes. Das and Ferbel proceed by defining a charge parity of $\eta_C(\gamma) = -1$ for a photon since the C operation reverses the electric field. This constrains the electromagnetic decay of a neutral particle like the $\pi^0$. The decay of the $\pi^0$ is:

$$\pi^0 \to \gamma + \gamma$$

This implies that the charge parity or behavior under charge conjugation for a $\pi^0$ is:

$$\eta_C(\pi^0) = \eta_C(\gamma)\eta_C(\gamma) = (-1)^2 = +1$$

Charge conjugation symmetry would imply that the $\pi^0$ will not decay by

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Particle concepts

References
Das & Ferbel
Ch. 11

Griffiths
Sec 4.7

$$\pi^0 \to \gamma$$

which we already know because it can't conserve momentum, but the decay

$$\pi^0 \to \gamma + \gamma + \gamma$$

can conserve momentum. This decay cannot happen because it would violate charge conjugation symmetry.

While the strong and electromagnetic interactions obey charge conjugation symmetry, the weak interaction does not. As an example, neutrinos are found to have intrinsic parities: neutrinos have left-handed parity and antineutrinos right-handed. Since charge conjugation would leave the spatial coordinates untouched, then if you operated on a neutrino with the C operator, you would produce a left-handed antineutrino. But there is no experimental evidence for such a particle; all antineutrinos appear to be right-handed. The combination of the parity operation P and the charge conjugation operation C on a neutrino do produce a right-handed antineutrino, in accordance with observation. So it appears that while beta decay does not obey parity or charge conjugation symmetry separately, it is invariant under the combination CP.

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# Time Reversal

Associated with the conservation laws which govern the behavior of physical particles, charge conjugation (C), parity (P) and time reversal combine to constitute a fundamental symmetry called CPT invariance.

In simple classical terms, time reversal just means replacing t by -t, inverting the direction of the flow of time. Reversing time also reverses the time derivatives of spatial quantities, so it reverses momentum and angular momentum.Newton's second law is quadratic in time and is invariant under time reversal. It's invariance under time reversal holds for either gravitational or electromagnetic forces.

Very sensitive experimental tests have been done to put upper bounds on any violation of time-reversal symmetry. One experiment described by Das and Ferbel is the search for a dipole moment for the neutron. Even though the neutron is neutral, it is viewed as made up of charged quarks and therefore could conceivably have a dipole moment. Experimental evidence is consistent with zero dipole moment, so time reversal symmetry seems to hold in this case.

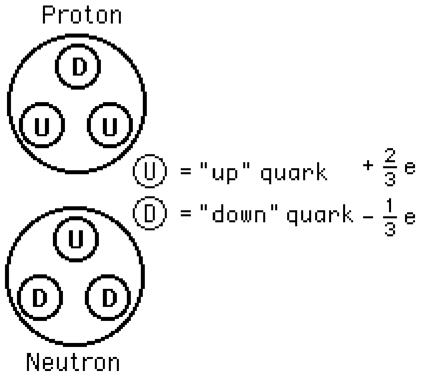
The small violation of CP symmetry suggests some departure from T symmetry in some weak interaction process since CPT invariance seems to be on very firm ground.

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

# The Eta Meson

| Particle | Symbol | Anti-particle | Makeup | Rest mass MeV/c^2 | S | C | B | Lifetime | Decay Modes |
|---|---|---|---|---|---|---|---|---|---|
| Eta | $\eta^0$ | Self | 2* | 548.8 | 0 | 0 | 0 | <10^-18 | $2\gamma$, $3\mu$ |




The fact that the free neutron decays

$$n \to p + e^- + \bar{\nu}_e$$

and nuclei decay by beta decay in processes like

$$P^{32} \to S^{32} + e^- + \bar{\nu}_e$$

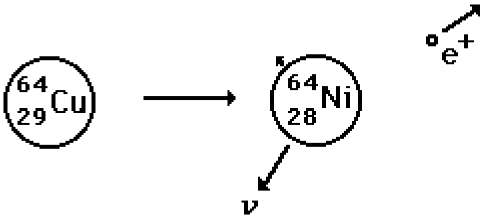
is thought to be the result of a more fundamental quark process

$$d \to u + e^- + \bar{\nu}_e$$

Table of quark properties

# Positron and Neutrino

The emission of a positron or an electron is referred to as beta decay. The positron is accompanied by a neutrino, a massless(?) and chargeless particle. Positrons are emitted with the same kind of energy spectrum as electrons in negative beta decay because of the emission of the neutrino.




# Relativistic Kinetic Energy

The relativistic energy expression includes both rest mass energy and the kinetic energy of motion. The kinetic energy is then given by

$$KE = mc^2 - m_0 c^2$$

This is essentially defining the kinetic energy of a particle as the excess of the particle energy over its rest mass energy. For low velocities this expression approaches the non-relativistic kinetic energy expression.

$$for \quad \frac{v}{c} << 1, \quad mc^2 - m_0 c^2 \approx \frac{1}{2} m_0 v^2$$ Show

For a mass of m = x10^ kg
= $m_e$ = $m_p$

with mass energy x10^ eV/c^2 = MeV/c^2 =
GeV/c^2

traveling at velocity v = x10^ m/s = c, gamma = γ =

the kinetic energy is

K.E.(relativistic) = x10^ joules = x10^ eV.
This compares to a non-relativistic result of x10^ joules ,
which would be in error by %.

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# Kinetic Energy for v/c<<1

The relativistic kinetic energy expression can be written as

$$KE = m_0c^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

and the square root expression then expanded by use of the binomial theorem :

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \ldots$$

giving

$$\left(1-\frac{v^2}{c^2}\right)^{-1/2} = 1 + \left[\frac{1}{2}\right]\frac{v^2}{c^2} + \frac{\frac{-1}{2}\left[\frac{-3}{2}\right]}{2!}\frac{v^4}{c^4} + \ldots$$

Substituting gives:

$$KE = \frac{1}{2}m_0v^2 + \frac{3}{8}\frac{m_0v^4}{c^2} + \frac{5}{16}\frac{m_0v^6}{c^4} + \ldots$$

$$KE \approx \frac{1}{2}m_0v^2 \quad \text{for } v<<c$$