Particle Physics – Properties of Quarks

In the earlier part of this course, we have discussed three families of leptons but principally concentrated on one doublet of quarks, the u and d. We will now introduce other types of quarks, along with the new quantum numbers which characterise them.

Isospin

It was noticed that many groupings of particles of similar mass and properties fitted in to common patterns. One way to characterise these is using isotopic spin or isospin, I. This quantity has nothing to do with the real spin of the particle, but obeys the same addition laws as the quantum mechanical rules for adding angular momentum or spin. When the orientation of an isospin vector is considered, it is in some hypothetical space, not in terms of the x, y and z axes of normal co-ordinates.

Nucleons (p, n), pi mesons (π^+, π^0, π^-) and the baryons known as Δ (Δ^{++}, Δ^+, Δ^0, Δ^-) are three examples of groups of similar mass particles differing in charge by one unit. The charge Q in each case can be considered as due to the orientation of an "isospin vector" in some hypothetical space, such that Q depends on the third component I_3. Thus the nucleons belong to an isospin doublet: $\mathrm{p} \equiv \left|I, I_3\right\rangle = \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle$; $\mathrm{n} = \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$. Similarly the pions form an isospin triplet, $\pi^+ = |1,1\rangle$; $\pi^0 = |1,0\rangle$; $\pi^- = |1,-1\rangle$. The Δ forms a quadruplet with $I = \tfrac{3}{2}$. The rule for electric charge can then be written $Q = e\left(\tfrac{1}{2}B + I_3\right)$, where B is the baryon number which is 1 for nucleons and the Δ and 0 for mesons such as the π. In terms of quarks, the u and d form an isospin doublet, $\mathrm{u} = \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle$; $\mathrm{d} = \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$ (both with $B = \tfrac{1}{3}$).

Three quarks with $I = \tfrac{1}{2}$ can combine to form $I_{\mathrm{tot}} = \tfrac{1}{2}$ or $\tfrac{3}{2}$. $I_{\mathrm{tot}} = \tfrac{1}{2}$ gives the nucleons while $I_{\mathrm{tot}} = \tfrac{3}{2}$ forms the Δ. In strong interactions, the total isospin vector (as well as I_3) is conserved. This is **not** true in electromagnetic or weak interactions.

Strangeness

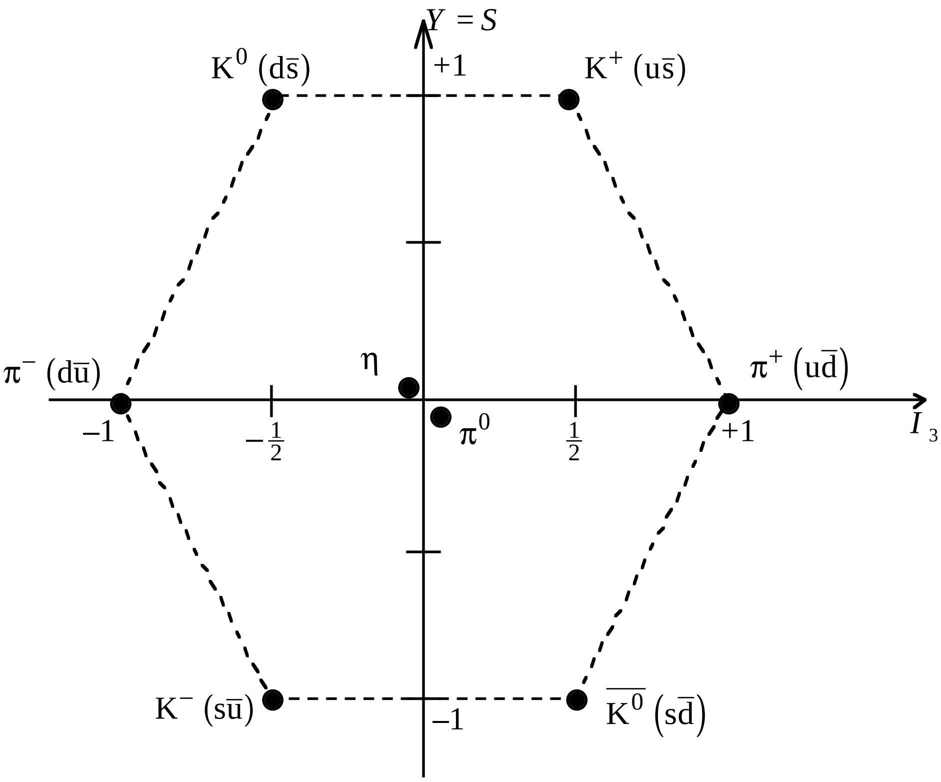
It was observed that some unstable particles produced in strong interactions had a long lifetime. This unusual stability for strongly interacting particles led to the term of strangeness. Such particles are always produced in pairs (associated production), and the quantum number of strangeness, S, was introduced, which is conserved in strong interactions. Thus in the interaction $\pi^- \mathrm{p} \to \Lambda^0 \mathrm{K}^0$, the Λ is assigned $S = -1$ and the K $S = +1$. The strange particles can only decay by the weak interaction, which does not conserve strangeness (as we will discuss later).

The formula for electric charge must know be modified to read

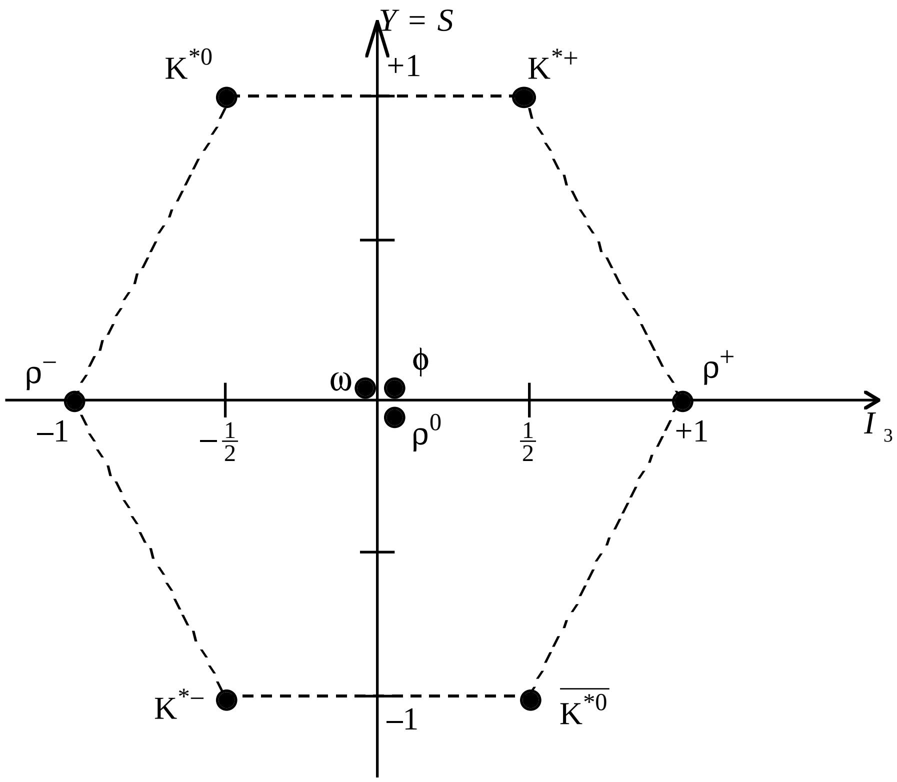
$$Q = e\left(I_3 + \tfrac{1}{2}B + \tfrac{1}{2}S\right) = e\left(I_3 + \tfrac{1}{2}Y\right)$$

where $Y = B + S$ is known as the hypercharge. (This formula is known as the Gell-Mann Nishijima relation.) Families of particles with similar properties (e.g. same spin and parity) can be plotted in terms of Y versus I_3, and form regular geometrical figures (see plots).

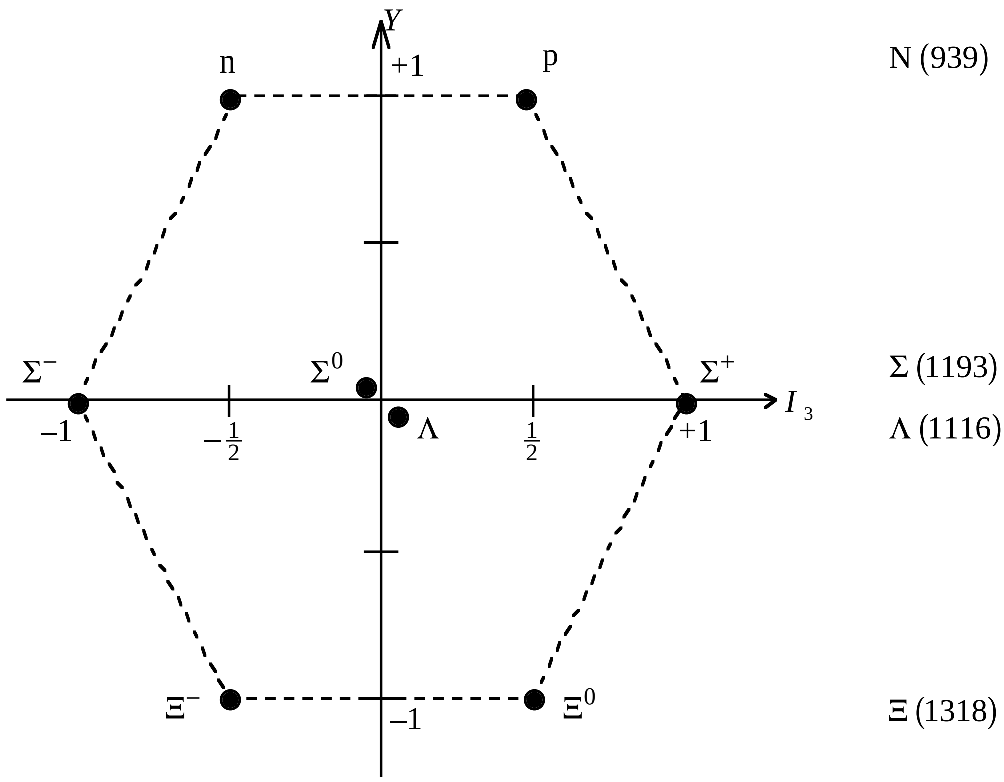
In terms of quarks we can introduce a new flavour of quark, the strange quark s. This has charge $-\tfrac{1}{3}$ and baryon number $\tfrac{1}{3}$ (like a d quark) but $I = 0$ and $S = -1$. It is also somewhat heavier than the u and d quarks. Since baryons consist of qqq, it is clear why no positive baryons exist with $|S| > 1$, while negative baryons are found with $S = -2$ or -3.



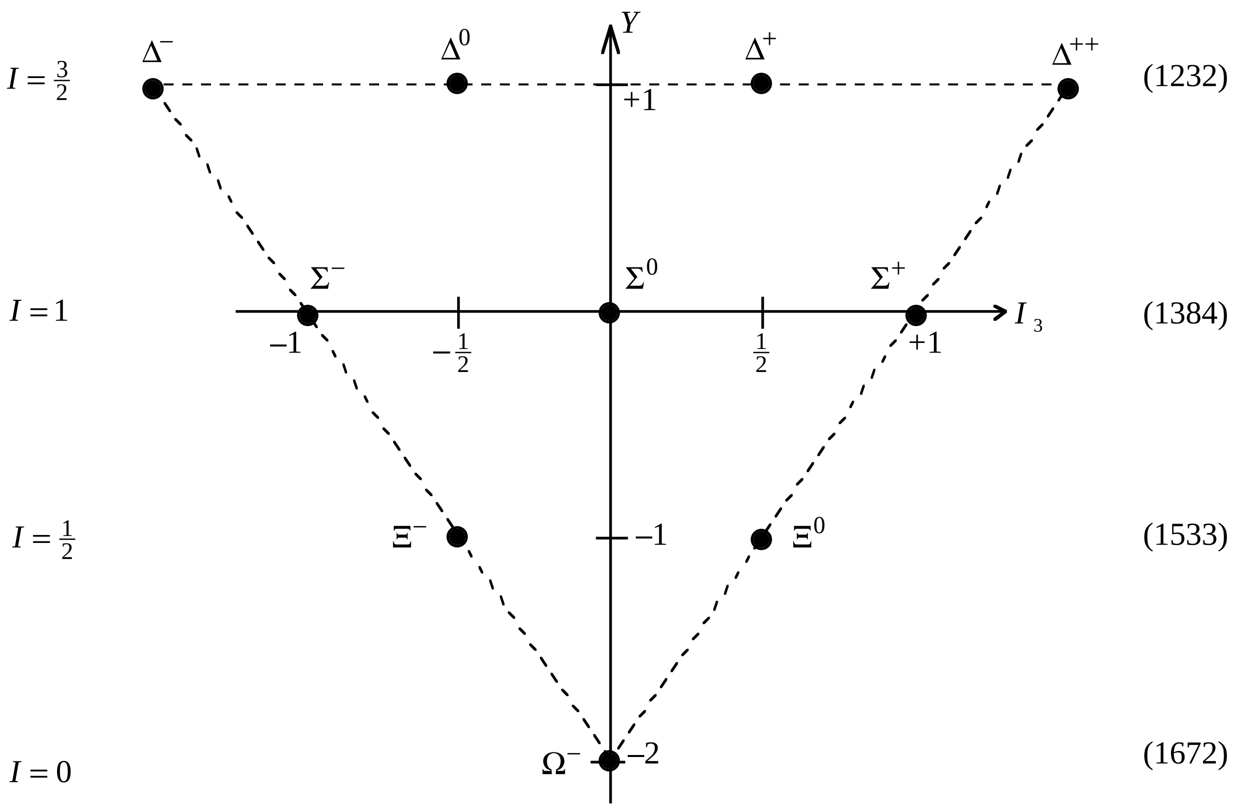
The lowest-lying pseudoscalar-meson states ($J^P = 0^-$), with quark assignments indicated.
(The states at the origin are displaced slightly for clarity.)



The vector-meson nonet ($J^P = 1^-$).
(Quark assignments are the same as above.)



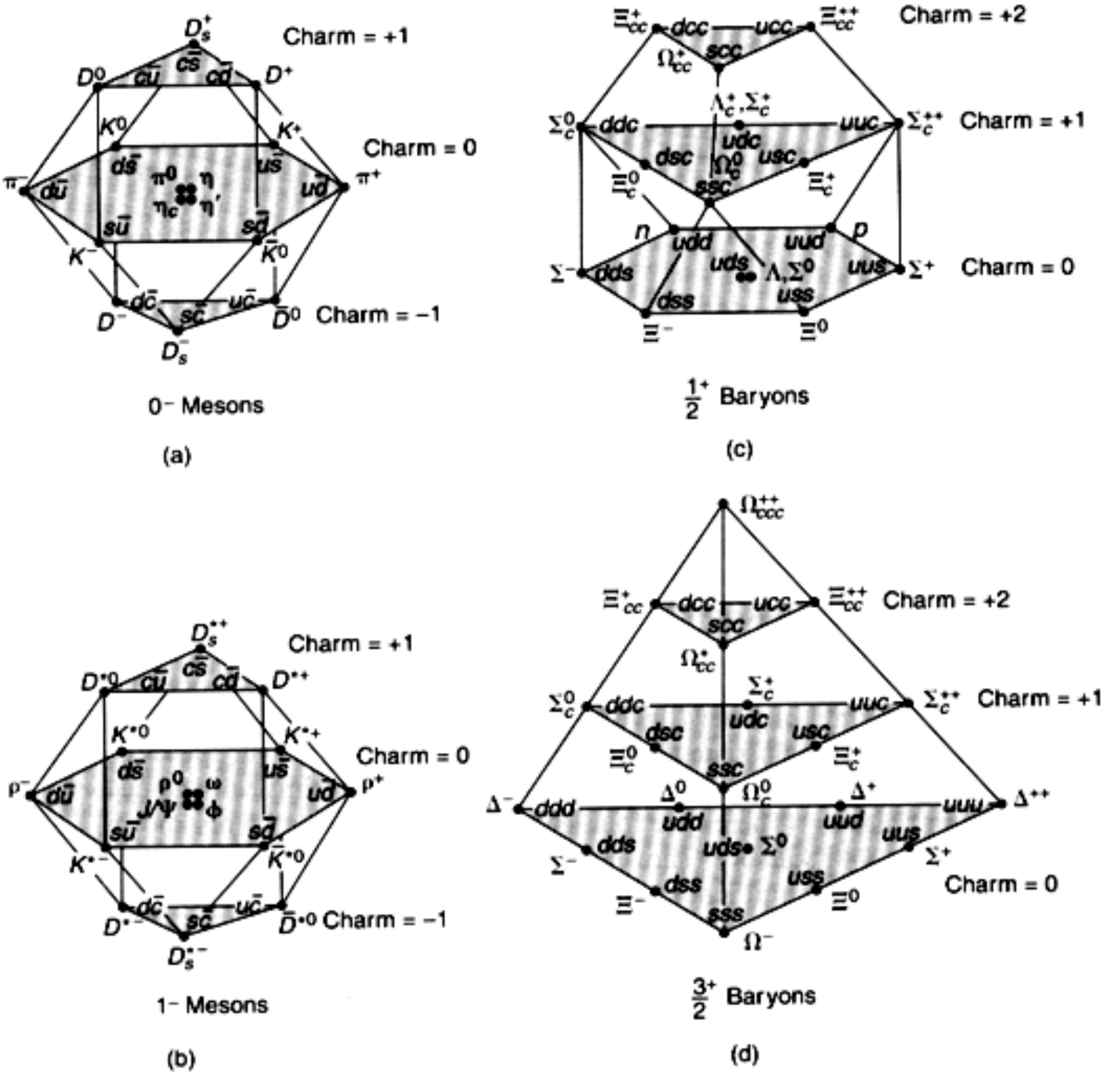
The baryon octet of spin-parity $J^P = \frac{1}{2}^+$



The baryon decuplet with spin-parity $J^P = \frac{3}{2}^+$

Further quarks

Other, still heavier quarks also exist. The charm quark, c, has a charge of $\frac{2}{3}$, like the u, and can be considered as a partner to the s. In 3 dimensions (see figure) particles containing c quarks can be plotted, and again show regular patterns.



Multiplets of hadrons containing up, down, strange and charm quarks. The slices through these figures where Charm = 0 correspond to the plane figures already shown in the previous diagrams, though containing new particles in the case of the mesons, composed of $c\bar{c}$.

We thus have 2 doublets or generations of quarks – (d, u) and (s, c). Since there are 3 doublets of leptons, there are theoretical reasons for expecting a third doublet of quarks too. Particles containing b quarks (bottom or beauty) were discovered in 1977. The b is an even heavier version of the d. Its partner, the t (top or truth) was first seen in 1994, and its mass has recently been measured at 174 GeV/c^2, with an uncertainty of about 10 GeV/c^2.

Quark Flavour and the Weak Interaction

As we have already seen, the strong and electromagnetic interactions conserve quark flavour, whereas the weak interaction may change it. In many weak decays, the changes are within a generation, e.g. in beta decay the W couples a u to a d quark; in the decay $D^+ \to K^0 \pi^+$ it couples a c to an s. However, this is not always the case, e.g. in the decay $K^- \to \pi^0 e^- \bar{\nu}_e$ the W couples an s to a u quark, and it was observed that such strangeness-changing decays were slightly weaker than strangeness-conserving weak decays.

Cabibbo explained this by proposing that the eigenstates of the weak interaction are different from those of the strong interaction. The strong interaction eigenstates are the u, d, s, c, b and t quarks, with well-defined isospin, strangeness etc. The eigenstates of the weak interaction, which does not conserve *I*, *S* etc., are said to be those of "weak isospin" *T*. For simplicity, let us first consider the first 2 generations alone. The weak eigenstates are the leptons and orthogonal linear combinations of the familiar quarks

$$\left\{\begin{matrix} T_3 = +\frac{1}{2} \\ T_3 = -\frac{1}{2} \end{matrix}\right\} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \begin{pmatrix} u \\ d_c \end{pmatrix} \qquad \begin{pmatrix} c \\ s_c \end{pmatrix}$$

with

$$d_c = \alpha\, d + \beta\, s$$
$$s_c = -\beta\, d + \alpha\, s \qquad \text{(normalisation } \alpha^2 + \beta^2 = 1\text{)}$$

α is usually known as $\cos\theta_c$, where θ_c is the Cabibbo angle. A value of $\sin\theta_c = 0.25$ is consistent with the observed apparent variation of weak coupling constant with reaction type.

The relationship between weak and strong eigenstates in 2 generations can also be expressed as

$$\underset{\textit{weak e-states}}{\begin{pmatrix} d_c \\ s_c \end{pmatrix}} = \underset{\textit{mixing matrix}}{\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}} \underset{\textit{strong e-states}}{\begin{pmatrix} d \\ s \end{pmatrix}}$$

The same formalism can be used for 3 generations, and the mixing matrix, known as the Cabibbo-Kabayashi-Maskawa or CKM matrix, can be parametrised in a number of ways.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underline{\underline{\mathbf{M}}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The magnitudes of the matrix elements have been determined experimentally, and are given with 90% confidence limits by

$$\underline{\underline{\mathbf{M}}} = \begin{pmatrix} 0.9745 \text{ to } 0.9760 & 0.217 \text{ to } 0.224 & 0.0018 \text{ to } 0.0045 \\ 0.217 \text{ to } 0.224 & 0.9737 \text{ to } 0.9753 & 0.036 \text{ to } 0.042 \\ 0.004 \text{ to } 0.013 & 0.035 \text{ to } 0.042 & 0.9991 \text{ to } 0.9994 \end{pmatrix}$$

Note that the values along the leading diagonal are quite close to one, those adjacent to it are significantly smaller, and the elements in the top-right and bottom-left corners are **much** smaller. This means that the mixing results in states which contain a small admixture of the quark from the next generation, while mixing between 1st and 3rd generation quarks is extremely small. Physically, this is revealed during weak decays in the relative probability of producing hadrons containing the respective quarks. For example, when a top quark decays it produces a b′ quark. This is bound in a hadron by the strong interaction, so must be revealed as a strong eigenstate. The b′ is most likely to result in a particle containing a b quark, with a smaller probability of an s quark and almost negligible likelihood of producing a d quark. Therefore, the diagonal structure of the CKM matrix means that weak decays are most likely to be <u>within a generation</u> if allowed by conservation of energy (a particle cannot decay into one that is heavier) or <u>to the next generation below</u> if this is not allowed. The most likely <u>overall</u> decay chain of a b quark is therefore $b \to c \to s \to u$.

For two generations, one parameter was required to describe the mixing. This was the Cabibbo angle. With three generations, 4 independent parameters are needed to define a general unitary matrix, and the individual matrix elements may have imaginary parts. One possible parametrisation of the CKM matrix is given below. Note that the following material is provided for completeness only, and is **not examinable**! (Further details are provided in the text books.)

$$\underline{\underline{\mathbf{M}}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with i and j being generation labels $\{i,j = 1,2,3\}$. In the limit $\theta_{23} = \theta_{13} = 0$, the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations, with θ_{12} identified with the Cabibbo angle. The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by suitable definition of the quark field phases. c_{23} is known to differ from unity only in the fifth decimal place.

If the parameter δ is non-zero, then the matrix is complex, and the small degree of CP violation present in the weak interaction can be explained naturally. This has not yet been proven!

[The above parametrisation and values are taken from the Particle Physics Data Booklet, from "Review of Particle Physics", European Physical Journal **C3**, June 1998, by the Particle Data Group.]