Light quark mesons

Now lets look at the bound states of the light quarks. The lightest quarks are u and d.

- **We will again combine quarks with anti-quarks to create mesons**
- **This time the bound states are highly relativistic, and hence we won't be able to use our "strong" potential to calculate the energy levels**
- **There are 2^2 =4 ways to put together u and d quarks**

Quarks	*Particle*	*Charge*	*Mass*	*Decay*	*Lifetime*
$u\bar{d}$	π^+	$+1$	$140\ MeV$	$\pi^+ \to \mu^+ \nu_\mu$	$2.6\times10^{-8}s$
$\bar{u}d$	π^-	-1	$140\ MeV$	$\pi^- \to \mu^- \bar{\nu}_\mu$	$2.6\times10^{-8}s$

- **The pion is practiacally stable! P ions are streaking through the atmosphere now**
- **π- and π+ are a particle anti-particle pair**
- **π- and π+ are the lightest charged mesons, so they can't decay via the strong or electromagnetic force, hence they decay via the weak force**

Neutral mesons with u and d

There are two more possibilities for putting together a u and d quark, both neutral

Quarks	Particle	Charge	Mass	Decay	Lifetime
$\frac{(d\bar{d} - u\bar{u})}{\sqrt{2}}$	π^0	0	135 MeV	$\pi^0 \to \gamma\gamma$	$0.8 \times 10^{-16} s$
$\frac{(d\bar{d} + u\bar{u})}{\sqrt{2}}$	η^0	0	548 MeV		

Something is wrong here

- **The π^0 and η wavefunctions are combinations of u and d quarks**
- **These are the "eigenfunctions" of the wavefunction with respect to the strong force.**
- **If you pulled a pion apart, half the time you would get a u-u pair, half the time a d-d pair**
- **The wavefunctions look similar to wavefunctions for the addition of two spin 1/2 particles**
- **The η mass is heavier than the π masses**

Adding two spin 1/2 states

Let's review the addition of angular momentum for two spin 1/2 states.

Consider the addition of two spin $1/2$ states

If there is no coupling of the two spins, we can treat them separately

$$S_{1z}|+\rangle_1 = \frac{\hbar}{2} \quad S_{1z}|-\rangle_1 = -\frac{\hbar}{2} \quad S_1^{\,2}|+\rangle_1 = S_1^{\,2}|-\rangle_1 = (s(s+1))\hbar^2 = \frac{3}{4}\hbar^2$$

$$S_{2z}|+\rangle_2 = \frac{\hbar}{2} \quad S_{2z}|-\rangle_2 = -\frac{\hbar}{2} \quad S_2^{\,2}|+\rangle_2 = S_2^{\,2}|-\rangle_2 = (s(s+1))\hbar^2 = \frac{3}{4}\hbar^2$$

The good quantum numbers are $S_1^{\,2}, S_2^{\,2}, S_{1z}, S_{2z}$

If there is a coupling between the spins $(S_1 \cdot S_2)$ then we need to work in a different basis, where the total spin $S = S_1 + S_2$ is the good quantum number. Introduce the notation

$|+\rangle_1 \otimes |+\rangle_2 \equiv |++\rangle$ the four possible combinations are

$|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$ in the old basis.

In the new basis, the states are eigenstates of $S^2, S_z, S_1^{\,2}, S_2^{\,2}$. The allowed value of S, S_z are in the notation $|S, S_z\rangle$ are

state	S	S_z	
$\lvert 1,1\rangle$	1	1	*Triplet*
$\lvert 1,0\rangle$	1	0	*Triplet*
$\lvert 1,-1\rangle$	1	-1	*Triplet*
$\lvert 0,0\rangle$	0	0	*Singlet*

These states are constructed from the old basis as follows

$$|1,1\rangle = |++\rangle$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle + |-+\rangle\right]$$

$$|1,-1\rangle = |--\rangle$$

Symmetric if +,- exchanged

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle - |-+\rangle\right]$$

Antisymmetric if +,- exchanged

The idea of Isospin

notice a similarity between the π wave functions and the addition of two spin 1/2s

- **As far as the strong force is concerned, the u and d quark are nearly identical particles**
- **We can think of them as being two different "spin" states of a generic quark**
- **This type of "spin" was given the name Isospin before the idea of quarks was invented**
- **Isospin is conserved in strong interactions (but not in weak or electromagnetic)**
- **The pion states form an isospin triplet formed from the two isospin 1/2 quarks**

We write the quark and anti-quark isospins as

$u = |+\rangle, d = |-\rangle$

$\bar{u} = |-\rangle, \bar{d} = -|+\rangle$

adding the two isospins together we get a triplet and a singlet

$$|1,1\rangle = |++\rangle = -u\bar{d} = \pi^+$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle + |-+\rangle\right] = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) = \pi^0$$

$$|1,-1\rangle = |--\rangle = d\bar{u} = \pi^-$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle - |-+\rangle\right] = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) = \eta^0$$

Adding the s quark

Historically a quantity called strangeness was also conserved in strong interactions.

- **Strangeness turned out to be due to the mesons with strangeness having a slightly heavier quark in them, this quark was named the strange quark**
- **With 3 quarks, we can have $3^2 = 9$ combinations of mesons**
- **The η mesons are a mixture of the three flavours of light quarks (u,d,s)**
- **The masses of mesons with s quarks are greater as the s quark is heavier than u,d**

Quarks	Particle	Charge	Mass	Decay	Lifetime
$u\bar{d}$	π^+	+1	140 MeV	$\pi^+ \to \mu^+ \nu_\mu$	$2.6\times10^{-8}s$
$\bar{u}d$	π^-	−1	140 MeV	$\pi^- \to \mu^- \bar{\nu}_\mu$	$2.6\times10^{-8}s$
$\frac{(d\bar{d}-u\bar{u})}{\sqrt{2}}$	π^0	0	135 MeV	$\pi^0 \to \gamma\gamma$	$0.8\times10^{-16}s$
$u\bar{s}$	K^+	+1	494 MeV	$K^+ \to \mu^+ \nu_\mu$	$10^{-8}s$
$d\bar{s}$	K^0	0	498 MeV	$K^0 \to \pi^+\pi^-$	$10^{-8}s$
$\bar{u}s$	K^-	−1	494 MeV	$K^- \to \mu^- \bar{\nu}_\mu$	$10^{-8}s$
$\bar{d}s$	$\bar{K}^0$	0	498 MeV	$\bar{K}^0 \to \pi^+\pi^-$	$10^{-8}s$
$\frac{(d\bar{d}+u\bar{u}-2s\bar{s})}{\sqrt{6}}$	η^0	0	549 MeV	$\eta^0 \to \gamma\gamma$	$\Gamma = 0.9keV$
$\frac{(d\bar{d}+u\bar{u}+s\bar{s})}{\sqrt{3}}$	η'	0	958 MeV	$\eta' \to \eta\pi\pi$	$\Gamma = 0.3MeV$

Spin and Parity of mesons

The 9 mesons we have constructed from the u, d, and s quarks have definite spin/parity

- **These are states with l=0**
- **The 9 mesons just listed also have their spins anti-parallel, so that the total spin=0**
- **In general, the parity of an anti-fermion is opposite to the parity of a fermion**
  - If the parity of a fermion is +
  - Then the parity of the anti-fermion is -
- **The parity of the meson is the product of the parity of the quark and anti-quark, hence the parity of mesons is (+)*(-)=(-)**
- **states are called pseudo-scalar mesons**

Parity (-) **Spin=0** **$\bar{q}q$ state**

The notation for mesons is J^P

$J^P = 0^-$ for the pseudoscalar mesons