

Light quark mesons

Now lets look at the bound states of the light quarks. The lightest quarks are u and d.

- We will again combine quarks with anti-quarks to create mesons
- This time the bound states are highly relativistic, and hence we won't be able to use our "strong" potential to calculate the energy levels
- There are $2^2 = 4$ ways to put together u and d quarks

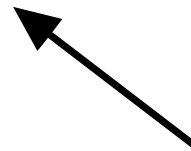
| <i>Quarks</i> | <i>Particle</i> | <i>Charge</i> | <i>Mass</i> | <i>Decay</i> | <i>Lifetime</i> |
|---------------|-----------------|---------------|-------------|---|------------------------|
| $u\bar{d}$ | π^+ | +1 | 140 MeV | $\pi^+ \rightarrow \mu^+ \nu_\mu$ | $2.6 \times 10^{-8} s$ |
| $\bar{u}d$ | π^- | -1 | 140 MeV | $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ | $2.6 \times 10^{-8} s$ |

- The pion is practically stable! P ions are streaking through the atmosphere now
- π^- and π^+ are a particle anti-particle pair
- π^- and π^+ are the lightest charged mesons, so they can't decay via the strong or electromagnetic force, hence they decay via the weak force

Neutral mesons with u and d

There are two more possibilities for putting together a u and d quark, both neutral

| Quarks | Particle | Charge | Mass | Decay | Lifetime |
|--|----------|--------|---------|----------------------------------|-------------------------|
| $\frac{(d\bar{d} - u\bar{u})}{\sqrt{2}}$ | π^0 | 0 | 135 MeV | $\pi^0 \rightarrow \gamma\gamma$ | $0.8 \times 10^{-16} s$ |
| $\frac{(d\bar{d} + u\bar{u})}{\sqrt{2}}$ | η^0 | 0 | 548 MeV | | |



Something is wrong here

- The π^0 and η wavefunctions are combinations of u and d quarks
- These are the “eigenfunctions” of the wavefunction with respect to the strong force.
- If you pulled a pion apart, half the time you would get a u-u pair, half the time a d-d pair
- The wavefunctions look similar to wavefunctions for the addition of two spin 1/2 particles
- The η mass is heavier than the π masses

Adding two spin 1/2 states

Let's review the addition of angular momentum for two spin 1/2 states.

Consider the addition of two spin 1/2 states

If there is no coupling of the two spins, we can treat them separately

$$S_{1z}|+\rangle_1 = \frac{\hbar}{2} S_{1z}|-\rangle_1 = -\frac{\hbar}{2} \quad S_1^2|+\rangle_1 = S_1^2|-\rangle_1 = (s(s+1))\hbar^2 = \frac{3}{4}\hbar^2$$

$$S_{2z}|+\rangle_2 = \frac{\hbar}{2} \quad S_{2z}|-\rangle_2 = -\frac{\hbar}{2} \quad S_2^2|+\rangle_2 = S_2^2|-\rangle_2 = (s(s+1))\hbar^2 = \frac{3}{4}\hbar^2$$

The good quantum numbers are $S_1^2, S_2^2, S_{1z}, S_{2z}$

If there is a coupling between the spins ($S_1 \cdot S_2$) then we need to work in a different basis, where the total spin $S = S_1 + S_2$ is the good quantum number. Introduce the notation

$|+\rangle_1 \otimes |+\rangle_2 \equiv |++\rangle$ the four possible combinations are $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$ in the old basis.

In the new basis, the states are eigenstates of S^2, S_z, S_1^2, S_2^2 . The allowed value of S, S_z are in the notation $|S, S_z\rangle$ are

| <i>state</i> | S | S_z | |
|----------------|-----|-------|---|
| $ 1,1\rangle$ | 1 | 1 | $\left. \begin{array}{l} \left[\begin{array}{l} \leftarrow \text{Triplet} \end{array} \right. \end{array} \right.$ |
| $ 1,0\rangle$ | 1 | 0 | |
| $ 1,-1\rangle$ | 1 | -1 | |
| $ 0,0\rangle$ | 0 | 0 | $\leftarrow \text{Singlet}$ |

These states are constructed from the old basis as follows

| | |
|---|--|
| $ 1,1\rangle = ++\rangle$ | $\left. \begin{array}{l} \left[\begin{array}{l} \leftarrow \text{Symmetric if } +, - \\ \text{exchanged} \end{array} \right. \end{array} \right.$ |
| $ 1,0\rangle = \frac{1}{\sqrt{2}}[+-\rangle + -+\rangle]$ | |
| $ 1,-1\rangle = --\rangle$ | |
| $ 0,0\rangle = \frac{1}{\sqrt{2}}[+-\rangle - -+\rangle]$ | $\leftarrow \text{Antisymmetric if } +, - \\ \text{exchanged}$ |

The idea of Isospin

notice a similarity between the π wave functions and the addition of two spin 1/2s

- As far as the strong force is concerned, the u and d quark are nearly identical particles
- We can think of them as being two different “spin” states of a generic quark
- This type of “spin” was given the name Isospin before the idea of quarks was invented
- Isospin is conserved in strong interactions (but not in weak or electromagnetic)
- The pion states form an isospin triplet formed from the two isospin 1/2 quarks

We write the quark and anti - quark isospins as

$$u = |+\rangle, d = |-\rangle$$

$$\bar{u} = |-\rangle, \bar{d} = -|+\rangle$$

adding the two isospins together we get a triplet and a singlet

$$|1,1\rangle = |++\rangle = -u\bar{d} = \pi^+$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}[|+-\rangle + |-+\rangle] = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = \pi^0$$

$$|1,-1\rangle = |--\rangle = d\bar{u} = \pi^-$$

$$|0,0\rangle = \frac{1}{\sqrt{2}}[|+-\rangle - |-+\rangle] = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) = \eta^0$$

Adding the s quark

Historically a quantity called strangeness was also conserved in strong interactions.

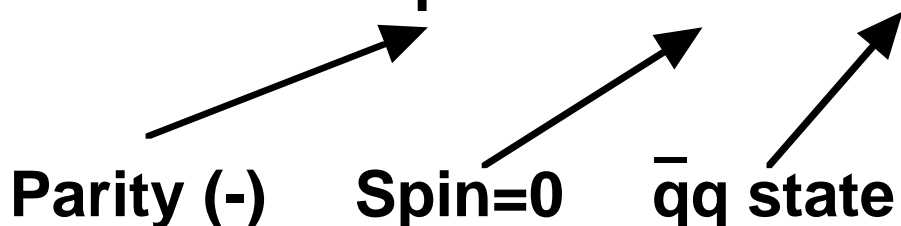
- Strangeness turned out to be due to the mesons with strangeness having a slightly heavier quark in them, this quark was named the strange quark
- With 3 quarks, we can have $3^2 = 9$ combinations of mesons
- The η mesons are a mixture of the three flavours of light quarks (u,d,s)
- The masses of mesons with s quarks are greater as the s quark is heavier than u,d

| Quarks | Particle | Charge | Mass | Decay | Lifetime |
|--|-------------|--------|---------|---|-------------------------|
| $u\bar{d}$ | π^+ | +1 | 140 MeV | $\pi^+ \rightarrow \mu^+ \nu_\mu$ | $2.6 \times 10^{-8} s$ |
| $\bar{u}d$ | π^- | -1 | 140 MeV | $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ | $2.6 \times 10^{-8} s$ |
| $\frac{(d\bar{d} - u\bar{u})}{\sqrt{2}}$ | π^0 | 0 | 135 MeV | $\pi^0 \rightarrow \gamma\gamma$ | $0.8 \times 10^{-16} s$ |
| $u\bar{s}$ | K^+ | +1 | 494 MeV | $K^+ \rightarrow \mu^+ \nu_\mu$ | $10^{-8} s$ |
| $d\bar{s}$ | K^0 | 0 | 498 MeV | $K^0 \rightarrow \pi^+ \pi^-$ | $10^{-8} s$ |
| $\bar{u}s$ | K^- | -1 | 494 MeV | $K^- \rightarrow \mu^- \bar{\nu}_\mu$ | $10^{-8} s$ |
| $\bar{d}s$ | \bar{K}^0 | 0 | 498 MeV | $\bar{K}^0 \rightarrow \pi^+ \pi^-$ | $10^{-8} s$ |
| $\frac{(d\bar{d} + u\bar{u} - 2s\bar{s})}{\sqrt{6}}$ | η^0 | 0 | 549 MeV | $\eta^0 \rightarrow \gamma\gamma$ | $\Gamma = 0.9 keV$ |
| $\frac{(d\bar{d} + u\bar{u} + s\bar{s})}{\sqrt{3}}$ | η' | 0 | 958 MeV | $\eta' \rightarrow \eta\pi\pi$ | $\Gamma = 0.3 MeV$ |

Spin and Parity of mesons

The 9 mesons we have constructed from the u, d, and s quarks have definite spin/parity

- These are states with $l=0$
- The 9 mesons just listed also have their spins anti-parallel, so that the total spin= 0
- In general, the parity of an anti-fermion is opposite to the parity of a fermion
 - If the parity of a fermion is +
 - Then the parity of the anti-fermion is -
- The parity of the meson is the product of the parity of the quark and anti-quark, hence the parity of mesons is $(+)*(-)=(-)$
- states are called pseudo-scalar mesons



The notation for mesons is J^P
 $J^P = 0^-$ for the pseudoscalar mesons